WHEN REMEMBERING AND PLANNING ARE WORTH IT: NAVIGATING UNDER CHANGE

Omid Madani* J. Brian Burns Reza Eghbali Thomas L. Dean

ABSTRACT

We explore how different types and uses of memory can aid spatial navigation in changing uncertain environments. In the simple foraging task we study, every day, our agent has to find its way from its home, through barriers, to food. Moreover, the world is non-stationary: from day to day, the location of the barriers and food may change, and the agent's sensing such as its location information is uncertain and very limited. Any model construction, such as a map, and use, such as planning, needs to be robust against these challenges, and if any learning is to be useful, it needs to be adequately fast. We look at a range of strategies, from simple to sophisticated, with various uses of memory and learning. We find that an architecture that can incorporate multiple strategies is required to handle (sub)tasks of a different nature, in particular for exploration and search, when food location is not known, and for planning a good path to a remembered (likely) food location. An agent that utilizes non-stationary probability learning techniques to keep updating its (episodic) memories and that uses those memories to build maps and plan on the fly (imperfect maps, *i.e.* noisy and limited to the agent's experience) can be increasingly and substantially more efficient than the simpler (minimal-memory) agents, as the task difficulties such as distance to goal are raised, as long as the uncertainty, from localization and change, is not too large.

"You can never know everything," Lan said quietly, "and part of what you know is always wrong. Perhaps even the most important part. A portion of wisdom lies in knowing that. A portion of courage lies in going on anyway."[2]

Winter's Heart, Book IX of the Wheel of Time, by Robert Jordan.

Alice remarked, "I can't remember things before they happen."
"It's a poor sort of memory that only works backwards," says the White Queen to Alice. [3]

Through the Looking Glass, by Lewis Carroll.

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1 Introduction

The rich, productive, and ever changing world necessitates agents capable of continuous and flexible adaptations to achieve their objectives. In the world of engineered AI systems, reinforcement learning (RL) techniques [55, 25], specially the model-free variety using deep feed-forward neural networks, have had substantial success in the past decade, as they are flexible, in that they do not assume much about the world, *e.g.* do not require modeling and encoding a complex world by the engineers of the agent, and powerful, as the (policy or value) function to be learned can be highly complex [49, 10, 37]. However, a critical limitation of the current model-free RL is the requirement of vast amounts of data. In dynamic real-world settings where the agent must adapt to evolving and changing tasks—ranging

*Correspondence to omidmadani@yahoo.com. The authors were partially affiliated with Brown University.

[2] Excerpt are from "The Bayesian Choice", by C. Robert [43] (thanks to Kevin Murphy for the book gift).

[3] We saw this first in G. Buzsaki's book, "The Brain from Inside Out" [8].

from minor adjustments to fundamental shifts—such systems often fall short: a change in the function (task) to be learned or performed can effectively reduce the available training sample size. Extensive and expensive simulations and substantial pretraining are common workarounds, striving to anticipate and train for *all possibilities*, as a way to enhance robustness, but the demands of the real productive world often curbs the success of such approaches. We seek efficient online learning and continual adaption that occurs ***in a lifetime, for a lifetime*** (*i.e.* keeping pace with life's changes and challenges).

Model-based techniques can potentially address some of the drawbacks of data inefficiency, their promise being that once acquired, the model(s) can be used repeatedly to find solutions for a diversity of related tasks. A challenge is what exactly an explicit model could mean, and whether such a model can be efficiently learned and updated. Consider spatial navigation, *e.g.* for foraging, which is a fundamental activity across the biological spectrum, with much research exploring how different minds meet its challenges [66, 50, 62, 4, 20, 29, 59, 38, 24]. Organisms need to be efficient (in energy/time) and flexible, and perform different but related navigation task (get to food, water, shelter, ...) in an at times dangerous and changing terrain (seasonal changes as well as abrupt changes, such as floods and droughts). The reward, from reaching the goal, is often distant, and the goal can change, thus the slow learning via reward propagation is often insufficient. From bacteria to bats and birds, living organisms utilize a range of sensing, memory, communication, and computational (e.g., inference) capabilities to efficiently reach their destinations [11, 59, 38].[4] While much of this machinery appears innate [29], a significant portion is likely dynamically learned and repeatedly tuned and reconfigured throughout an organism's lifespan. In particular, the hippocampus is a structure that is established to be critically involved in memory formation and use, for instance to help the agent navigate via the creation of the so-called *cognitive maps* (such as place cells and grid cells) [39, 59], though the details, such as what is represented and how such is used, continues to be debated and investigated [65, 41, 14, 51].

A map data structure, once built, is versatile and highly useful for navigation: at least theoretically, to get from any point A to any point B (on the map), one can plan using the map and execute the plan, *i.e.* one can solve a range of (related) navigation tasks rather efficiently when one has access to a good map. The map can be modified and reused if reroutings are sought (*e.g.* in case of new barriers blocking the routes that used to work). Consequently maps are the go to data structures for navigation tasks, *e.g.* in robotics and in particular SLAM domains [9]. But building a model (map) of a complex changing world, under limited time and sensing, carries its own many challenges. We explore these challenges in a simplified world and task, akin to the animats work [64, 53]: Consider a simple grid-world where every day an agent, with very limited sensing of its world, needs to find its way to food through barriers (road blocks). Moreover, the routes to food can change from day to day, some times substantially: several barriers or food may change location. A map could be part of a solution. If the world is fairly static, such a map could save much time over the lifetime of the agent. There are a number of challenges, including:

1. Any map learnt will be biased towards the experience of the agent, for instance, how much exploration it has performed (biased non-IID samples).
2. The world changes, and information extracted from the map can be out-dated (uncertainty 1).
3. Agent's knowledge of its current location (we use simple path integration) contains errors due to motion (action) noise (uncertainty 2)
4. How (and whether) an agent would carve and granularize its sensed and perceived space into locations, to serve its needs, remains open.

We explore the first three challenges in this work. Issues of world complexity and substantial uncertainty has thwarted the practical use of model-based techniques for open-ended real-world tasks that contain a diversity of uncertainties. Probabilistic planning is highly intractable in general [34, 30], and earlier works on agents have also found that the focus on explicit representations and planning in traditional AI approaches may be misplaced, in part to due to the aforementioned challenges, and in part because simpler agent strategies can be sufficiently successful in a diversity of worlds and tasks [2, 3, 7].

In this paper, we investigate a number of navigation strategies, from simple to the more sophisticated, to see how they compare as we vary certain aspects of task difficulty: the environment size (distance to food), the proportion of barriers (path complexity), and two types of uncertainty: daily barrier/food location change and the uncertainty of localization (in agent location). In particular, each agent type can use a mix of (pure) strategies to get to food, and we compare the more sophisticated agents to a fairly simple mixed greedy agent (Table 1), that uses random action selection some of the time, together with the strategy of (greedily) lowering its distance to goal, at other times. A fly, for example, may execute a strategy akin to greedy via the use of the smell sense [35, 20].

[4] Organisms of the same species, and the same organism but at different times, can use different (mix of) strategies [63, 23, 28]. The same person could use several strategies to get to a destination, *e.g.* from deciphering signs on a subway map to asking other people for directions (and remembering and executing those rough directions).

In general, the simpler strategies require less (of memory and computing machinery)[5] and can remain useful in a wider range of environments: In environments where food is abundant and near, and obstacles are sparse, they would be adequate. However, in harsher more challenging and richer environments,[6] and when there exists (sufficiently stable) structure to the world, and enough time to make a decision (think, reason, ...), the more sophisticated strategies may outperform the simpler ones. We have two goals in this work:

- We ask: *Are the more sophisticated map-based strategies worth their costs (of memory, intricate control, and in general compute machinery)?* Under what conditions are they better than the simpler ones? By how much?
- (upon a positive answer wrt their worth) We provide insights on agent architecture and the types of memory and learning that could support efficient map construction and effective updates (map maintenance), and map use.

We find that well-designed memory-based strategies, that appropriately take uncertainties into account, in building, updating, and using memories that ultimately serve as maps (in this paper), outperform mixed-greedy and other simple strategies, and the advantage grows, *e.g.* to over 20x fewer steps to food, with environment size and difficulty (distance to goal and barrier portion), as long as the rate of change and location-uncertainty is not too large. Due to uncertainty and task complexity, pure strategies, such as always planning for a goal, may underperform drastically, and we find that robust behavior needs to use several pure strategies to handle (sub)tasks of a different nature: search and exploration, when food location is unknown, and planning a good path or moving towards food when (likely) food location is known (Sect. 2.1). We describe an architecture where the agent uses one (active) strategy at any given time, but communicates the action taken and the sensory input to all strategies (so that memory-based strategies can update their memories appropriately). Furthermore, a planning-based strategy needs to take failure at planning time (*e.g.* no path to food) or execution failure (eg an unanticipated barrier) into account: Repeated (re)planning, as well as memory (map) updating, are necessities. Thus, there is indeed complexity to appropriate implementation of the more sophisticated strategies, but the gains in flexibility and reach can be worth it.

How does an agent (come to) know what to remember and how best to utilize its (episodic) memory? We assume certain capabilities, such as basic sensing, the importance of space and the details of path-integration, are (mostly) hard-wired [52, 13, 29]. In future work, we hope to reduce the number and the extent of the 'hard-wired assumptions' and in particular add additional learning in a lifetime.

This paper is organized as follows: We describe the simple grid environment and the basics of the agent and task(s) next. We then describe our flexible agent structure, which allows for incorporating and interfacing with multiple strategies (Sect. 3), followed by describing the (pure) strategies, with different uses of sensing, memory, learning, and plannings, in Sect. 4. The code is publicly available at GitHub [1]. Sect. 5 presents our experiments, and Sect. 6 discusses related work. We conclude in Sect. 7. A shorter version of this work appeared in BICA [33].

2 The Environment, the Agent, and Task(s)

Our environment is an N x N grid of N^2 cells, each cell identified by its location coordinates (x, y). There is a single agent, with limited sensing (to be described). The agent is in exactly one cell of the grid at any time, and can execute a (move) action to change its location by one cell. Time is broken into day and time tick: day $1, 2, 3, \cdots$, and within a day, time ticks $t = 1, 2, \cdots$. We focus on a simple closed-world: a cell is in one of three states at any time: **EMPTY**, **BARRIER**, or **FOOD**. The agent has four actions: move to a single adjacent cell without BARRIER (**UP**, **DOWN**, **LEFT** or **RIGHT**). With ***motion-noise probability*** p, for a low $p \in [0, 1]$ (*e.g.* $p = 0.02$) the environment picks an alternative 'noisy' position, and this includes staying in the same location and moving two steps forward (Fig. 2(b)). An **illegal** action is one leading to a barrier and has no effect. The agent cannot go off the grid (assume barriers). Upon action execution, time tick is incremented.

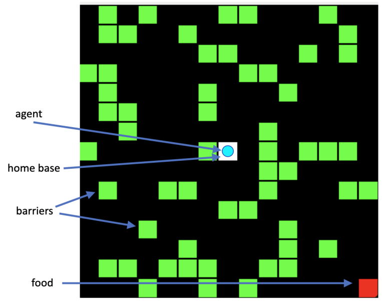
Activity in the environment is broken into daily trips: in our experiments, each day, the agent starts at the home base, $(0, 0)$, and the task of the agent is to get to the food location, and save steps in doing so.[7] The state of a cell is not changed within a day, and there is always exactly one food cell.[8], and we ensure that the generated environments are such that a path to food exists. Once the food is reached, the next day begins and the agent location is reset to home

[5]Simpler strategies may rely on more sophisticated sensing: sensing itself can be compute intensive and involve pipelines of processing as well, for instance for estimating any of localization, orientation, and wind direction *e.g.* for smell [21, 35, 20].

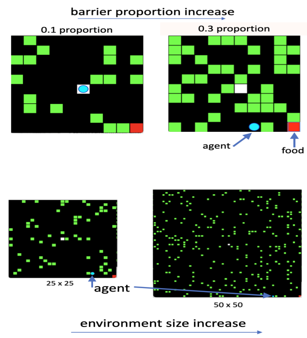
[6]One view in philosophy of biology posits that the organism (agent) itself, with its capabilities and interactions, determines its own environment [36].

[7]In this work, we ignore costs of computation (in particular planning) costs (time or energy), and assume any computation by the agent is performed within the budgeted time/computation bounds.

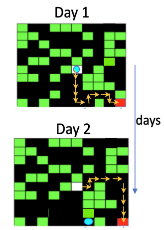
[8]One cell with food can reflect a region that has food in a more realistic setting.



(a) The task.



(b) 2 ways of raising task difficulty.



(c) Day to day food routes can change.

Figure 1: (a) The agent and its environment. It is important to emphasize that the agent *does not see* the whole grid, just the locations immediately adjacent to its current location (partial observability). (b) Two knobs on task complexity: barrier proportion and environment size (distance to goal). c) A 3rd knob on task difficulty: rate of (barrier) change, from day to day.

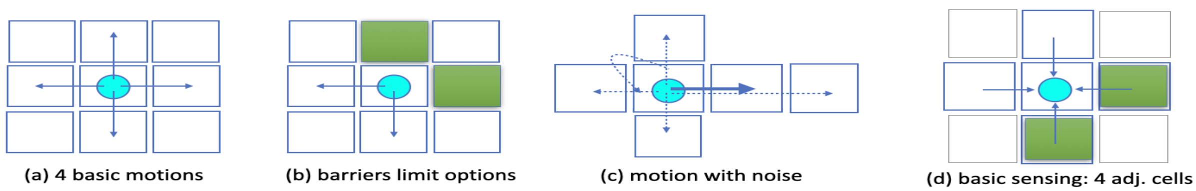


Figure 2: Basic actions and sensing: (a) 4 possible actions: LEFT(west), RIGHT, UP, or DOWN. (b) In this example, with two barriers, the agent has two legal actions (left and down). (c) Motion noise, up to 6 possibilities: when intending to go east (right), with some (noise) probability, the agent may end up in another location: stay in the same cell, go up, or down, or left, or go two hops east. (d) Sensing is also from a single adjacent cell (4 such).

$s = (0, 0)$. The food location does not change often, *i.e.* the food is often replenished, from day to day (so the strategy of trying to go back to the same place food was found tends to be useful). In most experiments, from day to day, we change several barrier locations, but keep the food in the same location (lower right corner).

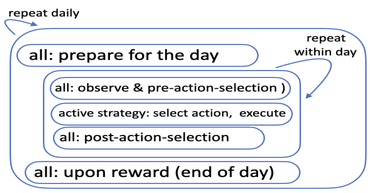
2.1 The Task(s): Getting to Food

A memory based agent deals with two tasks of a different nature in getting to food. Initially, on day 1, the agents doesn't know where the food is, nor the size of the grid, etc. (see the next section on limited sensing).[9] On the first day, and if (whenever) the food location changes, the task is more of a ***Search*** problem, *i.e.* search *in the environment*, for where food is, and strategies geared toward exploration are more successful. For an agent that doesn't have (episodic) memories, the task will always remain a search-in-the-environment problem. However, under some level of stability in the environment/task, agent could remember certain aspects during each day to help it better navigate and reach food in that day and in future days. Thus in subsequent days, the task may become more of a path **Planning** problem, *i.e.* search, but *internally*, using one's memories, for a good path (internal search). But this is only if the agent can remember the relevant aspects. Each agent type we experiment with uses a different mix of (one or more) basic strategies, and different strategies involve different types of memory (*e.g.* short within-day and long-term) and sensing (Sect. 3).

2.2 Change

From one day to next, several barriers may disappear and some new ones may appear. In our experiments, we use the (barrier) **change-rate** to set this change: a change-rate of 0.1 means that about 10% of the previous day's barriers are removed, and a similar number of new ones are added (overall barrier proportion kept the same from day to day). In one set of experiments, we also change the food location (Sect. 5.5).

[9] However, the strategies can be viewed as being designed for or having certain implicit/encoded assumptions in order to succeed in this type of task, such as 'it is useful to keep memories of what was observed at each location'.



Repeat (every day): // an agent's daily activity.
  Prepare for a new day: for each strategy, invoke its **new-day function**.
  Repeat until reward (*i.e.* food is reached)
    Get observations (interface with the environment)
    For each strategy, invoke its **pre-action-selection function**
    Select an action using the active strategy (**action-selection function**)
      If failure or times up, change strategy (round-robin & time budgets)
    For each strategy: invoke **post-action-selection function**
    Execute action (interface with the environment)
  End of day: for each strategy, invoke its **upon-reward function**

Figure 3: The control loop of a multi-strategy agent, responsible for the agent's daily activity. Each strategy has to provide an action-selection function, but other functions are optional. See Sect. 3.1 on how the agent changes its strategies (to find/reach goal), and Sect. 3.2 for the descriptions of the different functions.

2.3 Limited Sensing (Observing, Localizing, ..)

Animals use a variety of sense modalities for navigation, such as hearing (echo-location of bats), kinesthesis, olfactory, and vision. In our work, we support a few basic sense capabilities and different strategies may use a subset of them. One available sense is looking one cell adjacent/around to get its state (FOOD, BARRIER, EMPTY) (a visual radius of 1). For the greedy strategy, we assume the agent has a sense akin to smell, telling the agent which of the 4 actions reduces distance to goal.[10] All strategies know which actions are legal.

The more sophisticated strategies require **localization**: access to an estimate $(\tilde{x}, \tilde{y})$ of the true current location (x, y). Our agent performs simple **path integration** from its home $(0, 0)$, keeping two counters (sums), one for the horizontal, another for the vertical dimension. A RIGHT increments the horizontal counter, a LEFT decrements it (*e.g.* $(0, 0)$ becomes $(-1, 0)$), and so on.

Note that with a positive motion-noise p, the inaccuracy of the path-integration estimate $(\tilde{x}, \tilde{y})$ during the day is expected to grow with time tick t and in general the farther the agent is from its home base (starting point).

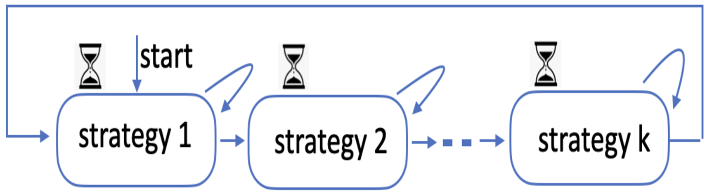
Dimensions of Difficulty. In some experiments we change the barrier portion or the grid size to change the difficulty of the task. For instance, a higher barrier portion means longer and more intricate paths to food, and remembering where barriers are or the successful past paths can become more useful. On the other hand, increasing the uncertainty, in our setting the barrier change-rate or the motion noise, can counteract the benefits of remembering.

3 Agent Structure

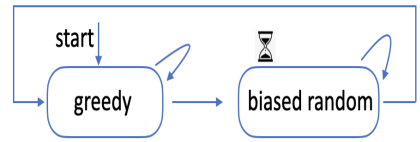
An (autonomous) agent is a system that senses and acts in an environment so that it reaches or satisfactorily maintains certain internal states.[11] This is reaching food for us. For a good review of the meaning of "agents" (and autonomy) see Franklin et al [17, 16] as well as Wilson et al [64, 53]. Figure 3 shows the basic loops for sensing and action selection and remembering (updating, learning, ...). We also make a distinction between an agent and a strategy. Briefly, a strategy provides a choice of action to the agent when queried by the agent, and given certain information such as the latest sense data as well internal states such as memories. Sect. 4 describes a range of strategies. An agent can be composite, *i.e.* use multiple strategies. Even in our fairly simple task, we have found that stand-alone pure strategies rarely work (in a plausible diversity of environments): the agent using a purely random motion is highly wasteful of moves, and the pure greedy agent can quickly get stuck in dead-ends (such as corridors with no outlets). Sometimes the agent needs to search for food (the Search task, Sect. 2.1), and at other times, the food location is known, and planning a path is a good strategy. Thus we seek 'composite' agents that use multiple strategies. In nature too, there is much evidence that organism use diverse strategies (eg allocentric or map-based vs sequential egocentric) [63, 23, 28]. Most our experiments use one or two strategies in an agent.

[10] We do not model noise in the greedy/smell direction (and one could increase such noise as the distance to food grows). On the other hand, smell can be more powerful and yield more information, such as the rough distance to goal, and in an extreme, one can imagine the barriers as impenetrable and tall, and the odor's rout to the agent translating to a path to the food.

[11] The internal states, such as the state of energy or rewards, are determined based on the agent's sensing as well.



(a) An agent can use several strategies in a round-robin fashion.



(b) The Greedy+Biased composite strategy.

Figure 4: An agent can use several strategies, or behavior modes, in a round-robin (fixed) ordered way in this paper (Sect. 3.1): (a) Each day the agent begins with using the first strategy. It moves to the next strategy (wrapping around), when current active strategy fails, or the strategy's time is up, until goal is reached (or all strategies fail). The allotted times are doubled each time it starts the list over in that day. (b) An example composite agent: The Greedy+Biased (mixed-greedy) agent begins with the greedy strategy and transitions to (biased) random in case of the failure of the greedy, and repeats this loop (each time, doubling the time allotted to random), until food is found.

However, our way of using multiple strategies is not 'decision making by committee': at each time tick, the agent takes the action selected by exactly one designated strategy, which we call the ***active strategy***. The agent sticks to its active strategy until it fails, or that strategy's current time budget is up, in which case the agent moves to the next strategy on the (user-specified) list of strategies (Fig. 4).

We begin by describing how an agent is structured in more detail next, in terms of how multiple strategies are arranged and interfaced with. This structuring makes it convenient to plug in various strategies and compare different composite agents.

3.1 Agent's Scheduling Logic and Progressive Time Budgets

Our agent uses its strategies in a **priority** order, in a **round-robin** fashion, and under **progressive (time) budgeting** to determine when to change strategy. The user specifies the ordered list of strategies to use, and the system goes over each strategy in the fixed priority-order given, and in a round-robin fashion. In the beginning of a day, at time tick 0, the highest priority strategy is used: the first strategy in the list is designated as the **(currently) active**. The trigger to change from one active strategy to next is either a **times up** or the case that the strategy **fails**. Failure for a strategy means that it does not return a (legal) action. For instance, for the case of the greedy strategy, failure means that no greedy actions is available due to barriers. This is an indication that some other strategy should be used. Some strategies, such as random, do not have a failure mode. Note that in general a strategy can become active (reactivated) multiple times in a day (due to the round-robin processing). If all strategies in the given list fail, the agent gives up too. In our experiments, we always include a strategy that never fails.

Progressive, in particular exponentially increasing, time budgets are motivated by the consideration that the agent in general does not know how much to stick with a given strategy (see greedy strategy, Sect. 4.2).[12] We use a simple dynamic budgeting technique: start the day with preset time limits, eg 1 unit (time ticks), and double the time limit whenever the strategy is visited again (is activated) in that day. This idea is akin to the exponential-backoff retries when a service does not work (temporarily) [44], *e.g.* in distributed systems, which helps avoid (network) congestion or overwhelming the service. Over the days, the agent could also learn aspects such as a better ordering of its given strategies or how much time to spent on each strategy. Again, such a learning technique must be responsive (adaptable) to changing environments.

In our comparisons, we create an agent, by specifying the ordered list of one or more strategies, often two, and the initial time budget, if any, for each strategy in the list (often either 0, meaning no budget, or 1).

3.2 Agent's Interfacing with the Strategies

Every strategy has to implement the mandatory action selection function, so that the agent can query it to get the action recommended by the strategy. In order to make a decision (*i.e.* provide the choice of action) a strategy may need various information, such as the action that was last executed, even if the strategy was not the active strategy (Sect. 3.1), and

[12]It is possible that the agent may have extra information, such as internal time (energy) budgets, and use such to inform and constrain its use of different strategies.

the state of various sense data. The agent provides this information via the following optional ***interfacing functions*** (agent↔strategy). Thus a strategy may implement up to 4 other interfacing functions (in addition to the mandatory action-selection function). The action selection function is invoked only for the active strategy, but these four are invoked (at appropriate times) for *all* the strategies of the agent:

1. The pre-action-selection function: As an example, a strategy could use this to record/remember what is observed around the agent.
2. The post-action-selection function: useful for recording/remembering what action was taken.
3. The beginning of the day function: useful for preparing for a new day (*e.g.* clearing certain memories, etc).
4. Once goal is reached, or upon-reward function: for recording the location of goal, etc.

These functions are optional, and for instance the random and greedy strategies implement none of the above. At every time tick, the agent first invokes every strategy's pre-action selection function, if any. After an action is selected, every strategy's post-selection function is invoked. The agent keeps and provides common information (such as the action taken, or the legal actions, current day, time tick, and estimated location, ...) to all the strategies. Because in our experiments, we end the day once food is reached, functions 3 and 4 above could be combined. Fig. 5 presents a summary of each strategy's requirements, such as localizing and memory.

3.2.1 Bypassing and Support for a Strategy Hierarchy

The above interfacing functionality can support certain *hierarchical* action selection patterns. For instance, if the food is seen next to one's cell, the agent should just go to it, and ignore (bypass) the active strategy's recommendation (which could be following an out-dated plan blindly): simply invoke the pre- and post-action selection functions of all its strategies. We have observed in our experiments that this bypassing capability lowers the number of steps to food somewhat. In this specific scenario, the 'lower level' *immediacy* bypasses the 'higher level', akin to reactive control [57, 26] (*e.g.* avoid imminent danger), while one can also imagine scenarios where the higher level should over-ride ("subsume") the lower level [6] (see section 6 in Kotseruba and Tsotsos [26]).

4 Strategies

A strategy provides a choice of action when queried by the agent. We next describe the strategies we experimented with, ordered based on extent of memory use (and complexity). See also Fig. 5 which presents a summary of each strategy's requirements (localizing, memory, etc).

4.1 Random Strategies

Our most basic strategy, **Random**, returns a legal move picked uniformly at random (up to four possibilities). It requires no memory, but is highly wasteful of steps. A small change, which we call ***Biased***, is a substantial improvement (Table 1). Biased does not take a 'step back' when possible, picking uniformly at random from the remaining legal actions (*e.g.* if LEFT was executed at $t-1$, RIGHT is not selected at t, unless it's the only legal action). This variant requires a bit of memory. There are other variants, such as adding a further bias to move forward when possible (in the same direction of the move at $t-1$), but our limited experiments did not show a clear benefit. The plain random strategy is simple and does not implement any of the pre and post interfacing functions. The Randomstrategy is a discrete random walk (the discrete version of the random Brownian motion), while Biasedand the next strategy can be viewed as forms of self-avoiding walks [66, 22].

4.2 Greedy Strategies

The Greedy strategy has access to the action(s) that lead to lowering the distance to the goal (up to 2 such), and picks one such at random. Greedy can fail, *i.e.* barrier(s) can block those directions, thus it cannot be used alone. Like Random, Greedy does not need any of the pre and post functions. A variant, **memory-greedy**, does not require a smell direction but requires the memory of the food location from the day before. If food is fairly static, it performs similar to greedy except when Search is required (*e.g.* on 1st day, Sect. 2.1).

4.3 Least-Visited (medium-term, or a day's, memory)

The **LeastVisited** strategy is our first strategy that makes extensive use of what could be viewed as a type of episodic memory (but only over a single day). This is also the first strategy that requires localization. This strategy, in its prepare-for-the-day function, allocates an empty mapping, of location to visited count (thus forgets yesterday's information),

strategy ↓	localization	smell	within day memory	multi-day memory	planning
Random	–	–	–	–	–
Greedy	–	✓	–	–	–
mem. Greedy	✓	–	–	✓	–
LeastVisited	✓	–	✓	–	–
Path	✓	–	✓	✓	–
ProbMap	✓	–	✓	✓	✓
DQN	depends	–	–	✓	–

Every day is a brand new day! (forgets experience)

Records/uses some memories, such as yesterday's food location.

Figure 5: A summary of what different strategies use or require (mainly of the agent, but also of the environment). Greedy requires the smell ('gradient') direction. Localization, ie availability of the $(\tilde{x}, \tilde{y})$ estimate of the current location for the agent, need not be perfect (Sect. 2.3). DQN's long-term memory is in its neural-network weights, and its input vector includes current $(\tilde{x}, \tilde{y})$ in our experiments.

and in its pre-action-selection function, increments the visit-count of its current location (location estimate, $(\tilde{x}, \tilde{y})$, via path-integration of Sect. 2.3). Whenever it is the active strategy, it picks an action that takes it to the cell with lowest visit-count (ties broken at random).

Compared to random, this strategy promotes more efficient exploration, and it is a useful strategy when (in effect) Search behavior is required, *e.g.* on the first day,[13] or whenever the food location has changed, and when other strategies (such as greedy) fail. Thus **LeastVisited** complements other memory or goal oriented strategies well.

4.4 Path-Memory (longer, over-days, memory)

The **Path** strategy remembers yesterday's path, in the form of a mapping, from visited location (state), $(\tilde{x}, \tilde{y})$, to action. There can be multiple actions performed at a given location, and the last action, selected by *any* active strategy, is what is remembered: performed in the post-action-selection function, and the previous recorded action for that location, if it exists, is over-written. If there is no change in the location of barriers nor food, and there is no motion noise, the remembered mapping is guaranteed to yield a successful path to food for the following day. This observation can be established fairly easily.[14] But note that the path may be significantly longer than necessary.

There are a few ***execution-time*** failure cases (akin to failures in the next planning section):

1. The (remembered) path, *i.e.* the mapping, returns an illegal action (*e.g.* due to a new barrier today)
2. The goal location is reached, but no food (*e.g.* due to localization error)
3. The location is not in the remembered path (from executing other strategies).

In case of failure, the agent changes strategy. It is possible that, for instance after exploring a bit, then landing on a later portion of the path, following the path would be useful again. Using the round-robin and progressive budgeting techniques allows for this possibility in a simple way (Sect. 3.1).

4.4.1 Relation to Model-Free RL

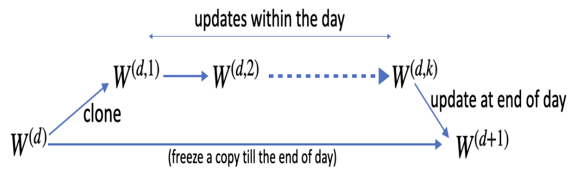
Path is akin to model-free RL solutions in that the path can be viewed as a policy, mapping locations (state features) to actions (also, akin to sequential egocentric [23]). This strategy works for only one goal, and if there are multiple goals or destinations (Sect. 5.5), the agent may need to learn different paths (different mappings, or functions), which can become both sample and space inefficient. We do not explore such extensions here. We also experimented with DQN [37], which is a model-free NN approach that has been very successful in fully-observable (Atari) game playing. We provided basic features (the surround) and location to the network (Sect. 5.4.1).

[13] Many organisms appear to have developed so-called Levy walk and jump strategies to more efficiently search a large expanse in finding clusters of food [66, 22]. LeastVisited in conjunction with planning (ProbMap) could be used for such search as well.

[14] Proof sketch: following the remembered mapping cannot end in a cycle, *i.e.* a node (location) being repeated. The out-degree of every location, as key in the mapping, is 1, and the indegree, for all nodes except the start node, can be shown to be exactly 1 as well (induction on number of hops from goal, and arguing no node can repeat in the induction step, as the agent reached the goal).

For each adjacent location s:
  For each of the episodic memories of s:
    Convert the memory to a memory-type m.
    Update performance of m (logloss).
    Update predictions of m.
  Store a new episodic memory for s.

(a) Pre-action-selection for ProbMap strategy.



(b) two-tiered, within-day and daily, updating.

Figure 6: (a) In the pre-action-selection function of the ProbMap strategy, for each of the 4 adjacent locations, predictions of memory-types are updated and a new episodic memory is formed. (b) A daily estimated statistic $W^{(d)}$, such as a predicted distribution by a memory-type, or its loss, is cloned at the beginning of the day, the original does not change during the day, while the other, within-day clone $W^{(d,1)}$, is used (*e.g.* for planning) and may be updated many times during the day yielding $W^{(d,k)}$. At the end of the day, the within-day $W^{(d,k)}$ is incorporated into $W^{d)}$ to generate the next day's daily statistic, $W^{(d+1)}$.

4.5 Probabilistic Map Strategy: Remember, Learn, Plan

The (probabilistic) map strategy, or **ProbMap** for short, records and keeps updating memories of the barriers and food, and uses such memories to make (in effect) a map, that includes a start and goal, and plans a path to the goal. The outcome of the planning, the computed path (plan) has a simple mapping (hashmap) structure, same as in Sect. 4.4 (a mapping from location to action). This strategy is the most elaborate, and the most expensive in terms of the compute and control infrastructure it requires, but once the memory is populated to a sufficient extent, and utilized as a map, it is the most powerful and flexible of the strategies, since the start and goal locations need not be fixed. The goals may not necessarily be food, *e.g.* exploratory unexamined destinations could be set as goals, though we do not explore such possibilities in this paper.

4.5.1 Making Memories

There are two types of memories that are maintained (recorded, updated, ...):

1. (episodic/fast recordings) What was observed at a given time and location.
2. (slow learning) For each ***memory-type*** (described below), learn and maintain its predictions (distributions) and prediction performance (we use logloss). These predictions are selected (based on performance) and used when planning.

As the agent navigates the grid, at each location $(\tilde{x}, \tilde{y})$, it updates its episodic memory for the four locations adjacent[15] $((\tilde{x}+1, \tilde{y}), (\tilde{x}-1, \tilde{y}), (\tilde{x}, \tilde{y}+1), \cdots)$: this is whether it saw FOOD, BARRIER, or EMPTY in each of the neighboring location (within radius 1). Memory updates are done in its pre-action-selection function (Fig. 6(a)). A hashmap, from (estimated) location $(\tilde{x}, \tilde{y})$ (location is key) to a list of such memory objects, is repeatedly updated. Each such memory is a record (object) that has a time field (day and time tick), and whether the agent saw a BARRIER, FOOD, or EMPTY at the location. For instance the episodic memory object <$(1, 2)$, BARRIER, 4> means BARRIER was observed on day 4 in location $(1, 2)$ (and is kept in the list of memories of location $(1, 2)$). This strategy keeps episodic memories from today, yesterday, and so on, kept up to some maximum number of days past, a parameter (*e.g.* up to 5).

Such an episodic memory can have different uses for inference, learning (long-term patterns), and ultimately prediction. For our task, in the ProbMap strategy, the memories are used for their predictive value when planning. They provide predictions on where food and barriers can be. This can be viewed as an instance of online supervised learning (different memory types predicting), and below we explain simple ways of computing and aggregating their predictions (for food and for barrier), although we expect much improvement is possible, which we leave to future work.

When there exists a memory of food (*e.g.* on day 2), this strategy constructs a map from its memories and plans a path from home to the remembered food location. It helps to extract probabilities, from the episodic memories (for barrier and food locations, described next). There are simpler non-probabilistic approaches: keep one memory for each location, the latest memory. The more elaborate method below allows for more flexibility and more efficient navigation (our experiments, in particular, when changing food locations, best reveal the advantages of keeping probabilities).

[15] The memory for the position the agent is currently on was updated in the previous tick, or is updated on the next tick.

4.5.2 Memory-Types serve as Predictors

Individual locations, or specific episodic memories, are not observed sufficiently often to provide reliable predictions (the problem of sparsity of specific, episodic, memories). Thus the ProbMap strategy maintains a predicted distribution for each *memory-type*. This is a kind of slow statistical learning (and online and supervised[16]), and a memory-type allows for an appropriate aggregation and generalization (addressing the sparsity problem). There are a number of ways of achieving this, and we describe one simple way, but alternatives should be explored.

In our implementation, a memory-type is specified by two components: time to current day, or the age of the memory, and the object that was seen (FOOD, BARRIER, and EMPTY). Time to current day is the number of days of the memory till today, so a memory-type from the current day (today) has 0 distance (to today), while a memory from the day before yesterday, yields a value (memory age) of 2.

Each memory-type predicts a 3-element distribution: a distribution is simply a probability value assigned to each of the three types of objects: EMPTY, FOOD, and BARRIER (the probabilities summing to 1.0). For example, assume BARRIER is observed today at location $(\tilde{x}, \tilde{y})$. Then, two days later, this memory corresponds to the memory-type <2, BARRIER>, and this memory-type provides a distribution for the same location $(\tilde{x}, \tilde{y})$.

In our implementation, at every time point, in the pre-action selection function (Fig. 6(a)), for each of the four adjacent locations, if there are episodic memories for that location, these memories are converted to memory-types and the logloss from their existing distributions (predictions) are updated, then the distributions (for each memory-type) is updated. Note that the same episodic memory is converted to a different memory-type on different days.

A memory could also learn and predict for other (more distant) locations, which we expect would be useful for this task, but we do not explore this possibility in this work.[17]. We leave exploring how memory types could be learned and further structured to future work.

4.5.3 Updating Predictions (Distributions)

We perform ***two-level*** updating (Fig. 6(b)), so that an atypical day[18] does not have a large adverse effect on the performance of a daily predictor for future days. In general, we want the within day prediction to adapt fairly fast to a day's observations and changes (in our case, localization errors),[19] while the model across multiple days to be more stable.

In the beginning of the day, each **daily** predictor is cloned to create a corresponding **within-day** predictor. The within-day predictor is used for prediction and planning in the day and is also continually updated. At the end of the day, the predicted distribution from the within-day predictor is used to update the daily predictor (which initializes the next day's within-day predictor).

Each predictor is a simple fixed window moving average predictor, where we use window size $K = 10$ for a within-day predictor, and $K = 5$ for a daily predictor. Each predictor keeps a queue of K 3-element distributions, and on an update, the oldest distribution, if at capacity K, is dropped, and a new distribution is inserted. These queuing methods are similar to those in [31], where predictors keep track of changing distributions, with a few differences: here, we've made a closed (and small) world assumption and, on the other hand, the input here is in general a distribution, not a single observed item at each time. Finally, in this task, we seek a good prediction even after one exposure (fast learning).

The within-day predictor, corresponding to a memory-type, works as follows. If it does not exist, it is allocated. Upon allocation, a memory-type predicts with its corresponding object with probability 1.0 (or near it).[20] For instance, the memory-type <2, BARRIER> initially predicts BARRIER with probability 1.0, or $[0, 1, 0]$, where we've taken EMPTY to correspond to dimension 1, and BARRIER and FOOD to correspond to dimensions 2 and 3 respectively. During the day such a memory type is updated several times. For instance, an observation of EMPTY inserts $[1, 0, 0]$ in the queue. Say in day 6 the agent moves to location (5, 3) and observes EMPTY in $(5, 4)$. The location $(5, 4)$ has memory <$(5, 4)$, BARRIER, 5>, which has type <1, BARRIER>. Thus the within-day predictor for <1, BARRIER> updates for the observation $[1, 0, 0]$. Later the agent moves to $(7, 7)$, and observes BARRIER in $(6, 7)$, and $(6, 7)$ also has memory <$(6, 7)$, BARRIER, 5> which again corresponds to memory-type <1, BARRIER>, and this predictor now updates for $[0, 1, 0]$. Say a new predictor with $K = 3$ observes the sequence [BARRIER, EMPTY, BARRIER,

[16]The observation of an item, such as FOOD or BARRIER, provides the supervisory feedback.

[17]This relates to how an agent may 'carve' its spatial world (see Sect. 1).

[18]For example when motion noise causes early and thus many subsequent errors in localization.

[19]These localization errors (independent action noise) do not necessarily apply to future days (independent errors). More generally, certain patterns in one day, eg weather events, may not generalize to future.

[20]More generally, the observation may have an associated probability (from the sensing/perception pipeline), and we could use such probabilities.

BARRIER, BARRIER, ...], then its predictions, after each update, would be the sequence $[0, 1, 0]$, $[1/2, 1/2, 0]$, $[1/3, 2/3, 0]$, $[1/3, 2/3, 0]$, $[0, 1, 0]$, ... Imagine the barrier change rate is 0.1 (for many days). Then memory-type <1, BARRIER> (a memory of a barrier from yesterday) is expected to predict around $[0.1, 0.9, 0]$.

4.5.4 Map Making: Sampling for Barrier Locations and Goal

ProbMap uses the distributions, provided by the within-day predictors (memory-types), to select a goal location (a destination to plan for) and to sample barrier locations.

In each planning session, the strategy needs to have a goal location. To set a goal, the ProbMap strategy first collects the list of all locations that could have food with sufficiently strong probability (above 0.01 in our experiments). For each location, the memory yielding the highest food probability provides the food probability for that location. If the list of food locations is empty (no location has sufficient probability, eg on day 1), no planning is possible, and the agent changes strategy (Sect. 3.1). Otherwise, and in each planning iteration (see next Sect. 4.5.5), the strategy picks a location from the food list with probability proportional to the normalized probabilities.[21] Note that since we use within-day predictors (for food and barriers), the probability of food (at a location) can change in a day, and the list of (food) destinations can shrink during the day.

We need barrier locations for planning too, and again we use sampling. For each location, there can be multiple episodic memories (from today, yesterday, etc). Each is converted to a memory-type. Some memory-types are better predictors than others. We use logloss to pick the best predictor and sample from its distribution to get whether there is a barrier at a location. If a location has no associated memory, the agent could use a prior, the overall chance of finding a BARRIER and EMPTY (which can be updated in every day and time step). In our current implementation, we assume empty (no barrier). This choice may lead to more exploration when planning. Due to uncertainty, change and limited exploration, this strategy, like Path, is not guaranteed to find the shortest path, but does relatively well (Sect. 5.4).[22]

In each planning iteration, a set of barrier locations is sampled and a goal (food location) is picked, and the strategy tries to find a path from start (0, 0) to goal, further described next.

Brief Discussion of Updating and Sampling. Food is sparse (in the environments of our experiments) and on the other hand important to the agent, and perhaps that explains the way we sample barriers (for planning) and food location (for setting a goal) are different: for barriers, we pick a best predictor, while for food, we use the highest probability prediction. It would be good to further clarify these underlying differences in addition to improving the updating (learning) and sampling techniques.

4.5.5 Planning a Path Given a Map (Constructed from Memory)

Whenever the ProbMap strategy is activated (such as the beginning of the day) and whenever there is a problem in plan execution (such as an unexpected barrier), the strategy performs planning, which, in case of success, yields a (new) path or plan to goal (same structure as the path agent). The strategy executes according to that plan/path until goal is reached, or until the next (execution-time) failure (described below) or times up. The planning algorithm is simple: pick and set a goal destination and extract (sample) a barrier map (Sect. 4.5.4), and use a search algorithm. We use A^* best-first search [19, 45].

There are two main types of failure:

1. Planning time failure, *i.e.* no food (no destination to go to, *e.g.* on day 1) or (seemingly) no path to food (within the time budget). The agent needs to move on to another strategy (Sect. 3.1).
2. Execution time failure, such as new barriers (see 4.4). The strategy replans in this case (unlike the Path strategy).

When planning, the strategy repeats path-finding (A^* search) a fixed number of iterations (5 in our experiments). As soon as one iteration yields a path, that path (plan) is returned. In each path-finding iteration, a goal location and barriers are sampled anew. A path-finding search aborts (stops that iteration) if the current path estimate exceeds the budget on number of steps (given to the strategy). Since barriers are sampled at the beginning of a planning iteration, a subsequent path-finding iteration may find a path (while previous ones failed).

[21] Thus, if there are two locations, one with 0.5 probability, and another with 0.7, then the first gets 0.5/(0.7+0.5), and the 2nd gets the remainder.

[22] In our current implementation, first memories are sampled, and a whole (barrier) map is constructed, then planning is carried out (the search for a path). An alternative implementation (possibly biologically more plausible) would access memories as needed during planning.

	mean-mean	med-mean	med-med	max-mean	max-max
Random	1965 ±617	1815	1200	3.5k	30K
Biased (Random)	536 ±163	479	329	1056	5.7k
Greedy+Biased (mixed-greedy)	200 ±110	167	37	541	3.3k
LeastVisited	250 ±49	252	198	395	1.5k
Greedy+LeastVisited	99 ±44	89	33	231	1.8k
Path+LeastVisited	209 ±48	203	155	325	1.7k
ProbMap+LeastVisited	80 ±35	79	35	230	1.7k
Oracle (theoretical minimum)	15.5 ±1.1	15.2	14	20.2	51

Table 1: Performance, *i.e.* the number of steps to goal (lower is better), of a range of strategies, from little or no memory, to extensive memory and computing (planning). The mixed ProbMap overall does the best (Oracle gives the minimum in the impractical case of complete knowledge). Experimental settings: 15x15 grids, 50 environments, 20 days each, 0.3 barrier-proportion, 0.1 change-rate, and 0.02 motion noise. Mean-mean is average number of steps over the 20 days and then average over the 50 initial environments, while med-mean is the mean of the (50) medians.

4.6 The Oracle Strategy

The Oracle strategy is meant to provide a reference point, the best (lowest) number of steps on a given day and environment. The strategy always has the complete up-to-date map (of barriers and food), as well as the true location of the agent, and plans accordingly (similar to ProbMap, uses A^* for planning, see Sect. 4.5.5). Its only limitation is that when motion-noise is present, the strategy may need to replan when the agent ends up in a location not in its current plan.

4.7 A Discussion of Types of Memory

There are many ways to classify memories, such as episodic, declarative, semantic, associative, working, short-term *vs.* long-term, internal *vs.* external (*e.g.* humans using notebooks as external memories), biographical, and so on. We noted that Biased uses a little memory, and control strategies such as round-robin and progressive need some memory for their operation. One aspect that distinguishes these from the memory used by LeastVisited , Path, and ProbMap is that the latter's episodic memory requirement grows in general with experience or the history of the agent (with the spatial expanse explored and/or over time), while the former (often control memory) is fixed (constant or near constant).

5 Experiments

We used PYGAME (www.pygame.org) to develop the environments and to visualize (code available on GitHub [1]). Our experiments involve 3 nested loops. With an outer loop of k_1 trials (*e.g.* 50), we generate initial environments (grids with certain barrier proportion).[23] In an inner loop of k_2 iterations we generate days: the initial environment is changed somewhat day after day (with a positive change-rate). Finally, within a day, the experiment continues for k_3 steps until the agent, starting from home, reaches food: k_3 depends on agent efficacy. We average k_3 over days and then over the environments, but also report daily medians (median-means: average the median over the environments) and maximums too (max-means), see Table 1. For instance, max-max refers to taking the maximum of the (k_3) steps over all the days and environments (the worst day). Each row of the table took seconds to complete on a Mac laptop.

For the composite agents, we used initial budgets of 5 steps on all strategies, doubling each time a strategy is reactivated (Sect. 3.1).[24] Greedy+Biased (or **mixed-greedy**) means start with Greedy then Biased in the round-robin fashion. The agent begins at the center of the grid, and the food is at the same corner on all the days, such as the lower right (except for Sect. 5.5).

We begin this section by discussing the results in Table 1 next. We then report results when changing parameters such as grid size (distance to food) and motion noise (comparing Greedy to ProbMap). We include further comparisons with the Path strategy as well as a model-free RL technique. Finally we look at performance when the goal location

[23] Beyond a barrier proportion of 0.3 often no path to food exists on 15x15 (under random generation), and so our experiments go up to the 0.3 level.

[24] In the shorter version of the paper [33], we used '0,1', *i.e.* no budget on Greedy, Path, and ProbMap (and initial budget of 1 for Biased and LeastVisited), which yielded somewhat fewer steps overall. However, ProbMap used a default initial budget of 50 in that case. To make the results more grid-size (distance to food) agnostic, we use initial 5 for all strategies.

is changed following a few patterns, and conclude with reporting on several statistics including the size of (episodic) memory consumed and the number of planning invocations (in a few settings).

5.1 Memory Helps!

Table 1 reports results on 15x15 grids, thus a good path should be close to 15 steps long, modulo motion noise and barriers, and the Oracle 's performance, 15.5 mean steps (the minimum possible), reflects that. We can see that as we go down the table (use strategies that roughly use increasing more memory), the performances, specially the worst cases, on very bad days (max-max), improve substantially. Biased, which is Random but with a bit of memory that prevents the agent from reversing (Sect. 4.1), does substantially better than Random.

The best of the memory-based strategies substantially beats Greedy+Biased, and in particular we get increased robustness. If we ignore the first day or so (the Search days), this gap grows (mean-mean of ProbMap improves to 50). Note that the expected performance of Greedy+Biased can not change from day to day (no remembering). The best combination (of smell-direction capability and memory), *i.e.* ProbMap+Greedy+LeastVisited , gets a mean of 56 (not shown in the table).

5.2 Interaction of Size, Change-Rate, and Motion Noise

Fig. 7 shows that when grid size or barrier portion difficulties are raised, with low motion-noise, the relative gain of ProbMap compared to Greedy grows, to over 30x (substantial gains, as distance grows, under no noise). Fig. 8 shows, on the other hand, that motion noise has the reverse effect, and at some point, Greedy can outperform. Increasing the grid-size can compound this.

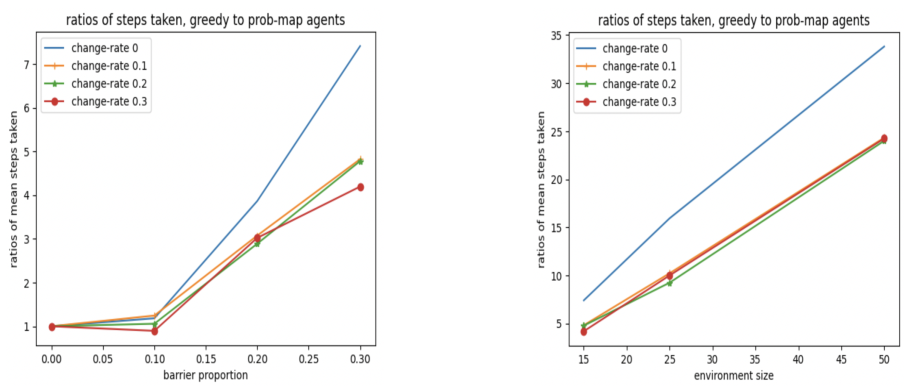


Figure 7: Efficiency gains, ProbMap+LeastVisited compared to Greedy+Biased, mean steps over 20 days and 50 environments. The ratio of steps taken, Greedy+Biased to ProbMap+LeastVisited, (a) as we increase barrier-proportion ratio, for different change rates (on 15x15, 0 motion-noise), and (b) as we increase environment size (from 15x15 to 50x50), keeping barrier proportion at 0.3.

5.3 Keep Updating (Continual Remembering, Learning, ...)

Fig. 9 shows that even though the performance of ProbMap+LeastVisited may appear to plateau, the agent needs to keep remembering and learning under (barrier) change to preserve its performance (*e.g.* similar findings on the need for continued updates in [31]). Note that the performance of the strategies that do not use episodic memories (*i.e.* Random, Greedy, and LeastVisited) does not change (improve) over days.

5.4 Further Comparisons with the Path Strategy

The Path variant trails the Greedy variants in the setting of Table 1. If we lower the motion noise and change rates to 0, Path gets a mean-mean of 39 beating 70 for Greedy+LeastVisited (and ProbMap gets 18).

ProbMap is more robust to change compared to Path. Neither approach is guaranteed to find a shortest path, but as Table 2 shows, under 0 change and motion noise, ProbMap+LeastVisited tends to find significantly shorter paths. Interestingly, as we increase the barrier proportion, the relative under performance of the Path variant (ProbMap+LeastVisited) first shoots up, then as more barriers are added goes down again.

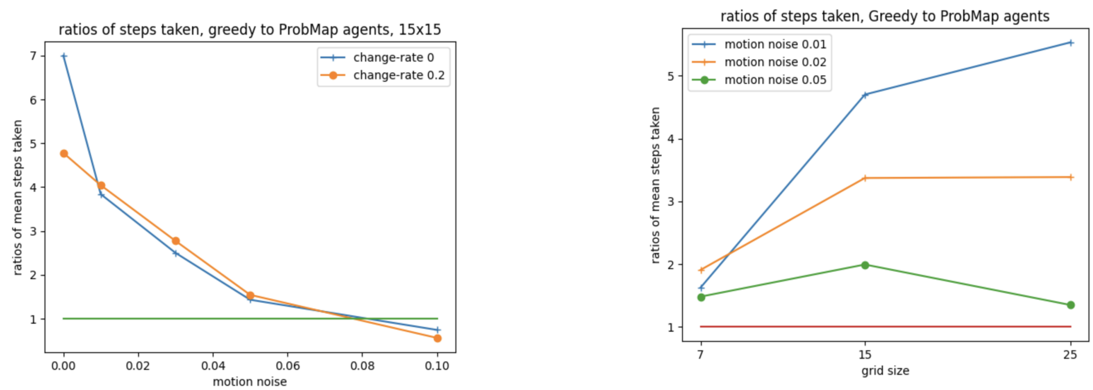


Figure 8: As motion-noise is increased, the advantage of ProbMap+LeastVisited compared to Greedy+Biased degrades. Left: on a fixed 15x15 grid (barrier proportion 0.3), Right: as we increase the grid size (distance to goal). However, with 0 motion-noise, the performance advantage grows with distance (as in Fig. 7).

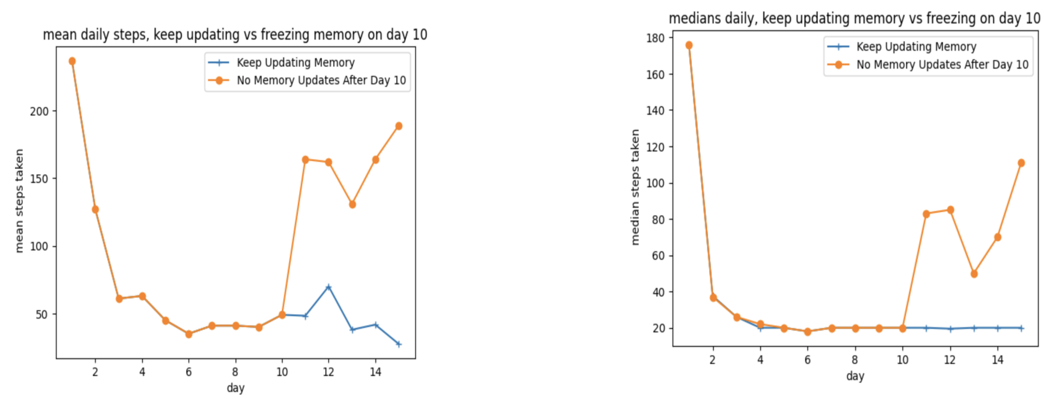


Figure 9: Continual learning (memory updates) vs freezing memory (*i.e.* stopping updates after day 10). Same setting for Table 1 except for 150 environments. Daily steps, means (left) and medians (right), as a function of day (each point is the mean or median, 150 values, of number of steps taken to food on that day).

5.4.1 A Model-Free NN-Based Strategy

We also experimented with DQN [37], a model-free RL technique which performs very well on a range of (fully-observable) Atari games. The agent is given the location, and the surrounding barrier and food information (radius 1, as other agents). On 15x15 grids, and no barriers nor uncertainty (*i.e.* no change in barriers and no motion noise), the agent learns a good path eventually: a learning rate of 0.002 works best (compared to 0.01 and 0.001),[25] and yields a mean of 80 (median 40) for 20 days, and mean of 30 after 50 days, and 20 for 200 days. However, as we increase barrier proportion, *e.g.* to 0.1, the learning becomes unstable, and on some environments the number of steps even after several days is in the 10s of thousands, occurring frequent enough that we could not often run such on 20 environments. If we set the change rate to 0.1, the fraction of environments when the agent hangs (has many days with say over 50k steps) goes up. The DQN agent needs both to find the reward, and to pass that information gradually to other cells nearby, and implicitly learn that barriers block. All this, reward propagation, takes times (many steps and days), and if there is change (in goal or barriers), issues of learning stability arise.

5.5 Changes in Food Location

Recall that the episodic memories recorded, of barriers and food, have a time field too (the day and tick is recorded), which can be helpful for discovering when there is a pattern to the changes in food or barrier locations. In this section, we explore how the ProbMap strategy, using such memories, handles certain changes, either random or systematic, to

[25] Other parameters: batch size of 5, gamma of 0.9, eps_start of 0.9, end of 0.05 with decay of 10. We also experimented with changing those parameters.

mean & max steps	on 15x15				on 30x30			
barrier prop. →	0.0	0.1	0.2	0.3	0.0	0.1	0.2	0.3
Path+LeastVisited	28, 28	44.6, 110	35.2, 68	28.7, 58	62, 62	91.4, 206	92, 254	67.3, 172
ProbMap+LeastVisited	14, 14	14.5, 24	15.8, 46	16.4, 52	28, 28	29.2, 52	31.7, 66	36.8, 130
Oracle	14, 14	14.1, 16	14.6, 20	16.0, 36	28, 28	28.0, 30	28.5, 36	32.3, 76

Table 2: The average and maximum (max-max) number of steps to goal on 15x15 (left) and 30x30 (right), on 50 environments, average and max taken over the last 10 days of 30 days in each environment, as barrier proportion is increased, with 0 change-rate and 0 motion noise. ProbMap+LeastVisited remains much closer to the performance of the Oracle. Interestingly, the performance of Path is not monotonic with increasing barrier proportion (unlike the other two strategies): it first goes up (degrades) then comes down (improves).

↓ memory kept	$k = 2$, round-robin	$k = 2$, uniform	$k = 4$, round-robin	$k = 4$, uniform
1 day	178	124	182	179
2 days	28.4	81.1	192	165
5 (default)	34.7	59.2	33.1	159
10 days	29.7	55.3	41.4	129

Table 3: The performance (mean-mean) of ProbMap+LeastVisited as food location is changed from day to day (Sect. 5.5): k=2 means over 2 corner locations, and k=4 is over 4 (15x15 grids, change-rate of 0.1, barrier-portion of 0.3, motion noise of 0, 50 environments, 30 days) and averages are over the last 10 days (of 30 days). Oracle and Greedy+Biased, both being insensitive to the food location pattern, get respectively around 15.5 and 160s in this setting. Keeping episodic memories and memory-types helps the agent to discover the food pattern lowering its step counts. When more memory is kept, the agent can handle larger k.

food location, from day to day. In these experiments, the food is placed on one of a fixed list of k corner locations: $k \in [2, 3, 4]$ and the list is not changed for the duration of the experiment. In one version of the experiments, “uniform”, the food location is picked uniformly at random from the list. As always, the agent is not given any information on food location. Here, to perform well, a memory-based agent needs to remember likely places where food could be. In another set of experiments, “round-robin”, we change the food among the k in a strict round-robin manner, thus when $k = 2$, the food oscillates from one corner (odd days) to the other (even days). In the round-robin cases, the predictiveness of an agent’s food memory locations will not be monotonic with time: when $k = 2$, the memory from two days ago is better than the food memory from yesterday (recall that each memory-type only provides, or predicts, a distribution for its own location in our current techniques, Sect. 4.5.2).

Table 3 summarizes the performance of ProbMap+LeastVisited , when we change the memory capacity of ProbMap from the default of 5 days (on 15x15 grids with barrier portion of 0.3). We observe that keeping only one day underperforms drastically in all cases, while keeping 2 improves a bit, in particular for the oscillating $k = 2$ case (where exactly two memories from past 2 days is sufficient), but insufficient for the uniform $k \geq 2$. We note that, in general, the uniform case is harder than the oscillating case (in these experiments), as older memories may be needed to best keep track of the possibilities of where food may be. If the agent keeps only a few days’ worth of memory (if it doesn’t stretch long enough into past), a run of several consecutive days where the food is always in one corner completely erases the possibility (knowledge) that the food could be on another corner as well, and when the agent doesn’t find the food in the expected corner, the problem becomes a Search problem.

We note that, as in Sect. 5.3, the agent is indeed learning. For instance, for the case of $k = 2$ round-robin (oscillate), if we average the number of steps on the first 10 days, we get (mean) over 80 steps, and if over first 5 days, we get 130 steps. Finally, to compare to Table Sect. 1, if we also introduce 0.02 noise in motion, the mean steps goes up to around 100 (averaged over last 10 days). Finally, as expected, our Path strategy (designed for a single goal) does not do well in these multiple goal experiments, getting a mean of over 200 steps (average over last 10).

5.6 Brief Exploration of a few other Statistics

We go back to the setting of Table 1 (unless specified), in particular one corner food location, and focusing on the ProbMap+LeastVisited strategy, and report statistics on a few other aspects of that strategy.

budgets →	progressive (5,5)	5	10	20	40	60	80
Greedy+Biased	200	–	263	127	164	209	229
ProbMap+LeastVisited	80	89	92	83	94	118	117

Table 4: The default ('5,5') progressive budgeting (first column) *vs.* different fixed time budgets (*e.g.* the 2nd column is fixed budget of 5 on both Greedy and Biased). The setting is as in Table 1, and mean-means are reported. Progressive budgeting allows for a robust way of using multiple strategies. Greedy+Biased does not finish with a fixed budget of 5 (Sect. 5.6.1).

change-rate and motion-noise →	0, 0	0.1, 0	0, 0.02	0.1, 0.02
ProbMap+LeastVisited 15x15	1, 309, (90, 20)	3, 357 (90, 31)	3, 433 (100, 59)	5.5, 517 (101, 58)
ProbMap+LeastVisited 25x25	1, 545, (274, 41)	5, 722, (274, 86)	7, 1.7k, (376, 241)	10, 1.7k, (365, 267)

Table 5: Under a few change-rate and motion noise levels, the median number of planning invocations (computed over days 2 onwards), the mean number of episodic memories at day 20 (middle number), and the number of cells in the visit counts on days 1 and 2, are shown (environment parameters as in Table 1 unless specified). As the task gets harder, more memory is required and additional replannings are carried out (Sect. 5.6.2). The visit-count goes down significantly on day 2, specially with low motion noise (with a more focused path).

5.6.1 Progressive Budgets

Progressive budgeting doubles the time budget of a strategy (if not unlimited) each time the strategy is activated in a day (Sect. 3.1). This is a convenient way of flexibly increasing the time allocation without knowing how far the food is or the level of the (barrier) complexity of the environment. As Table 4 shows, setting the budget to a fixed value that is too small or too large can lead to too many steps: with fixed budget of only 5 on both Greedy and Biased, the corresponding greedy agent gets stuck (switching to greedy too early leads to heading back to a dead end again and again). The table's results are on 15x15 (same setting of Table 1. On 30x30 (not shown), progressive budgeting's mean performance for the greedy agent is often double that of well-chosen fixed-budgets, while for the ProbMap agent, the performances remain comparable. Thus there is likely room for significant improvement: for instance, learning good beginning of day budgets, from what levels worked on previous days, as well as perhaps tuning the multiplier (rather than the default of 2), may improve the results. We also note that at times there can be a trade off between mean *vs.* median performance (in number of steps): while the median number of steps may go down (*e.g.* when lowering the budget of exploration), the average (and maximum steps over the days) may go up.

5.6.2 Memory Consumption and Replannings

When ProbMap+LeastVisited is used, planning is invoked at the beginning of the day (except for day 1) and every time the current plan fails (*e.g.* a new barrier) and whenever the strategy is reactivated in a day. We expect the number of plannings to increase as change rate or motion noise are raised. Table 5 shows this for 15x15 and 25x25, confirming our expectations. The table also shows memory consumption: the number of episodic memories, on day 20 (averaged over 50 environments), as well as the visit counts, on days 1 and 2.[26] Since we keep up to 5 days-past worth of memory, and each memory corresponds to one location, the maximum episodic-memory consumption is up to $6 \times 25 \times 25 \approx 4k$.

We observe that increasing motion noise significantly increases memory consumptions as well, and the visit counts can be significantly lower on day 2 and subsequent days (following a path), compared to day 1 (search for food).

5.6.3 State of Memories (Remembered Food, etc)

With 0 motion noise, the number of remembered food locations stays at 1 on days 2 onwards, irrespective of (barrier) change-rate or grid-size, as expected. With motion noise of 0.01, the remembered food locations goes to 2 to 4 on subsequent days on 15x15 (on day 3 it can be up to two locations with positive probability, etc), and with motion noise of 0.02, we observed 5 or 6 (*e.g.* on day 10). When we increase grid-size to 25x25 (distance to food), there is more opportunity for error in localization, and at motion noise levels 0.01 and 0.02, we get many more 5s and 6s locations than for 15x15 (up to 5 past days memories by default). An example evolution of food locations with the probabilities day by day is shown in Table 6. Note that a memory may not necessarily get updated during a day (when not observed), specially that FOOD is sparse, and we keep up to 5 days memory by default. As food is important and sparse, it may be good to have a differential policy as to which memories are kept longer.

[26] All these statistics are reported in the end or start of the next day function (*e.g.* before the visit counts are reset for next day).

day 2: ((13, 16), 1.00)
day 3: ((14, 15), 1.00), ((13, 16), 1.00)
day 4: ((14, 15), 1.00), ((13, 16), 1.00), ((14, 14), 1.00)
day 5: ((14, 15), 1.00), ((13, 16), 1.00), ((14, 14), 1.00)
day 6: ((14, 15), 1.00), ((13, 16), 1.00), ((14, 14), 1.00)
day 7: ((14, 15), 1.00), ((14, 14), 1.00)
day 8: ((14, 14), 1.00)
...
day 12: ((14, 14), 0.80)
day 13: ((14, 14), 0.97), ((15, 14), 0.96)
day 14: ((14, 14), 0.96), ((15, 14), 0.95)
...

Table 6: Locations and food probabilities at the beginning of each day (on a run on 15x15, 0.02 motion noise). The real food is on a single location with true coordinates of (14, 14) (see Sect. 5.6.3).

With a barrier portion of 0.3 and change-rate of 0.1 on 15x15 and 0 motion noise, the memory type <1, BARRIER> (*i.e.* BARRIER was seen yesterday at a location x), at end of day 2, predicts BARRIER (at same location x) with probabilities that are mostly 1.0 and sometimes 0.8 (expected 0.9, and the remainder probability going to EMPTY). By end of day 20, these probabilities show a larger numeric diversity, but, with more experience, less variance and closer to 0.9 (a few runs: 1.0, 0.76, .80, 0.92, ...). And <2, BARRIER> (from two days ago) has a lower precision (probability for BARRIER) as expected (expected 0.81), at day 20 (a few runs): 1.0, 0.52, .56, 1.0, 0.8, ... With a higher change-rate of 0.3, at end of day 2, a few probabilities for BARRIER, given by <1, BARRIER>, are: 0.4, 1.0, 1.0, 0.2, ..., and at end of day 20: 0.64, 0.8, 0.6, 0.72, ... (expected 0.7). Similarly for <1, EMPTY> (which predict EMPTYwith a higher probability, and other memory types such as for food. And as we add motion noise, the imprecision grows. Note that these probabilities are not only a function of how many samples have been observed, but also the biased towards the paths that the agent takes.

6 Related Work

Our work is related to diverse tracks of research on learning and decision making, including the nature and use of memory (in biological and artificial systems), reinforcement learning (RL), planning, cognitive architectures, autonomous agents, and robotics. We focus on closely related work that was not discussed earlier.

Two broad behaviors or strategies have been identified in computational neuroscience: model-based (*e.g.* map making, and goal oriented) *vs.* habitual instrumental behavior (corresponding to our Path strategy and typical model-free RL): The evidence and the relative strengths and weaknesses are discussed in [12] (such as the simplicity of the habitual *vs.* the flexibility but the computational requirements of goal-oriented model-based behavior). Human memory representations are complex and are sufficiently flexible to have a diversity of uses, *e.g.* not just for spatial maps, but, for instance, also for the more general cognitive graphs (diverse uses) [41, 42]. Unsupervised learning of (explicit) structured representations, that would find repeated diverse use, may also be foundational for perception [32].

In a partially observed (limited sensing) task [61], the authors study how (predictive) memory and RL techniques could be put together in a perceptually realistic navigation setting, showing promising results that the agent using predictive memory was substantially more successful than plain model-free RL agents (*e.g.* remembering and finding the way to goal when teleported), but the number of episodes (environment steps) remained considerable (*e.g.* 100s of thousand or millions) and generalization ability remains unclear. In a recent study [60], the authors tackle the tricky question of LLM generalization, and develop novel evaluation techniques (*e.g.* querying under small perturbations) for a few tasks. For a navigation task (in Manhattan, NY), they present evidence that the (transformer) generative network does not learn a systematic ('coherent') map from its sequence training data, *e.g.* not generalizing as well to sequences that are perturbed from the shortest paths sequences used for training.[27]

Recent work, motivated by the possibility of map construction by humans, experiments on subjects as well as performs computational modeling providing evidence that humans build map structures consistent with a Bayesian approach [48], and furthermore, such maps help planning via partially observed Markov decision processes (POMDPs), in a continual

[27]The perturbation to action sequences, by which the authors study network performance in one of their experiments, is akin to our random barrier location changes, and to a lesser extent to our motion noise: in their work, the agent (the transformer) is given the alternative move taken, and is queried for a plausible next move.

(re)planning fashion. Our study focuses on how repeated change and uncertainty could help or limit the benefits of learning a map, in a simple daily agentic foraging task, and the environment and interaction durations are parameterized (attributes such as grid size and uncertainty characteristics can be changed substantially) and we compare a variety of strategies (pure-sensing or smell-based, and path-memory techniques).

Change, in machine learning and RL, continues to be studied and remains a challenge, and techniques in areas such as continual learning, transfer learning, distribution shift, meta learning, lifelong learning, lifelong RL, and open-ended learning in robotics attempt to address the various dimensions of the problem [58, 47, 40, 5, 56, 46]. The issue of environment change facing animals was also studied in [67] and the authors propose that animals achieve change detection and change of strategy in part through counter-factual reasoning (as typical RL does not completely explain the observed speed of change in behavior). See also work on replay [15]. In the area of simultaneous localization and mapping (SLAM) for robotics, richer perception and change is also identified as a future area of research [9].

The work on cognitive architectures explores how components such as perception, memory, and control can be combined, inspired by research in human and animal intelligence, with the goal of shedding further insight onto human intelligence as well as striving to create general problem solvers with applications to areas such as robotics [26, 18, 27]. Our work began with a more narrow focus (memory and navigation), but consideration of different environments and somewhat different tasks (Search *vs.* Plan) also extended our final solution to be somewhat more general than originally anticipated. Our work also shares goals with research on artificial animals, so-called *animats*, in that we study intelligence in the context of an agent with limited sensing with the goal of sustaining itself [64, 53, 54]. In particular, Strannegård *et al.* advocate for agent architectures and learning mechanisms for a diversity of situations (worlds and agent needs, supporting multiple objectives, and different sensing/motor capabilities), and explore dynamically growing memory (learning) structures [53]. In that work, the patterns learned are deterministic and the feedback (reward/cost) was not delayed significantly. We focused on navigation under change and uncertainty as well as the use of (episodic) memory.

7 Summary and Future Directions

Organisms have limited access to external worlds that are uncertain, complex, and non-stationary. This includes changes in the environment, *e.g.* in the weather (temporary, periodic, or permanent change) as well as in one's capabilities (*e.g.* losing an arm, or gaining new skills). We focused on a closed but changing world, and on developing agents that can remember and learn fast (learning along a life trajectory, keeping pace with change), and accepted that certain aspects of the strategies that the agent deploys are hardwired (*e.g.* geared towards the navigation task), while other aspects could be learned and tuned continually. We showed that an agent with significant memory and computing (planning), with appropriate algorithms and architecture, can substantially out-perform a greedy-smell agent (purely sense-based with no episodic memory) as long as change and uncertainties are not too large.

We touched on a variety of future directions throughout the paper. In general, we hope to reduce the number and extent of the 'hardwired' assumptions we made, towards more autonomy. For instance, how could an agent come to know what to remember (and how to use it), and how does it 'carve' and granularize its spatial and temporal inputs? Related to this is support for richer perception as well as *open worlds*, such as learning and using various environmental and task regularities in an *agent friendly* manner, *i.e.* sample efficient and robust to limited and biased experience, continually adapting, and cumulative when possible.

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